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ISSUE**

SEPTEMBER 2004  
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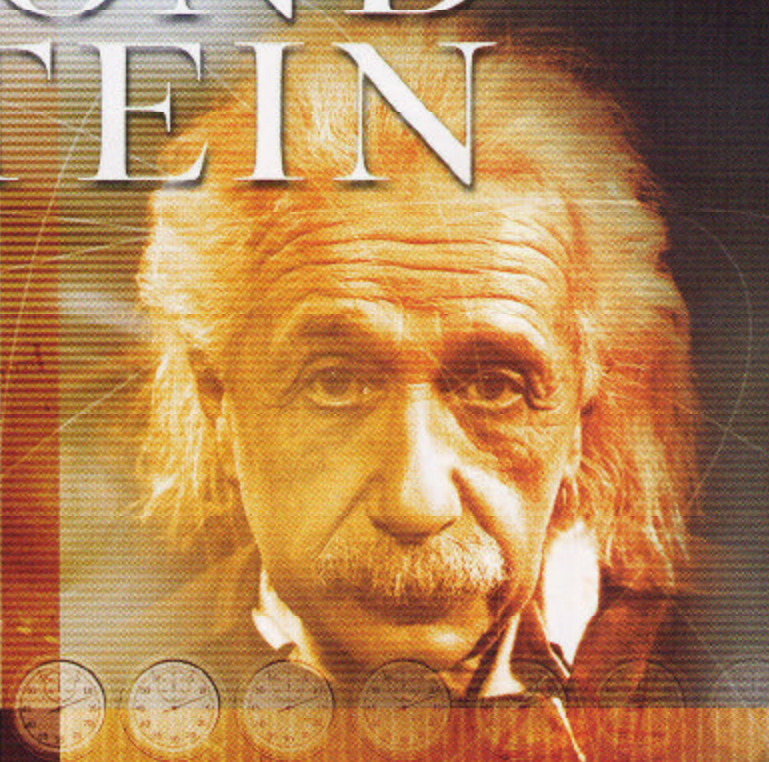
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# A COSMIC CONUNDRUM

A new incarnation of Einstein's cosmological constant may point the way beyond general relativity

By Lawrence M. Krauss and Michael S. Turner

## In 1917 Albert Einstein faced a confusing problem

as he tried to reconcile his new theory of gravity, the general theory of relativity, with the limited understanding of the universe at the time. Like most of his contemporaries, Einstein was convinced that the universe must be static—neither expanding nor contracting—but this desired state was not consistent with his equations of gravity. In desperation, Einstein added an extra, ad hoc cosmological term to his equations to counterbalance gravity and allow for a static solution.

Twelve years later, though, American astronomer Edwin Hubble discovered that the universe was far from static. He found that remote galaxies were swiftly receding from our own at a rate that was proportional to their distance. A cosmological term was not needed to explain an expanding universe, so Einstein abandoned the concept. Russian-American physicist George Gamow declared in his autobiography that “when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life.”

In the past six years, however, the cosmological term—now called the cosmological constant—has reemerged to play a central role in 21st-century physics. But the motivation for this resurrection is actually very different from Einstein's original thinking; the new version of the term arises from recent observations of an accelerating universe and, ironically, from the principles of quantum mechanics, the branch of physics that Einstein so famously abhorred. Many physicists now expect the cosmological term to provide the key to moving beyond Einstein's theory to a deeper understanding of space, time, and gravity and perhaps to a quantum theory that unifies gravity with the other

## OVERVIEW

- Quantum mechanics and relativity, combined with recent evidence of an accelerating universe, have led physicists to resurrect the cosmological term that Einstein introduced and later repudiated. But this term now represents a mysterious form of energy that permeates empty space and drives an accelerated cosmic expansion.
- The efforts to explain the origin of this energy may help scientists move beyond Einstein's theory in ways that are likely to change our fundamental understanding of the universe.

LONELY UNIVERSE may be our ultimate fate if the cosmic expansion keeps accelerating—a phenomenon believed to be caused by the cosmological constant. The orange spheres represent the observable universe, which grows at the speed of light; the blue spheres represent an expanding patch of space. As expansion accelerates, fewer galaxy clusters are observable.



fundamental forces of nature. It is too soon to say what the ultimate resolution will be, but it is likely to change our picture of the universe.

### Birth of a Constant

GENERAL RELATIVITY grew out of a decade-long struggle by Einstein to follow up on his pivotal observation in 1907 that gravity and accelerated motion are equivalent. As expressed in Einstein's well-known thought experiment, the physics inside an elevator sitting at rest in a uniform gravitational

field of strength  $g$  is exactly the same as the physics inside an elevator that is rocketing through empty space with a uniform acceleration of  $g$ .  
that was finite, static and adhered to Mach's principles (for instance, a finite distribution of matter trailing off into emptiness did not seem to satisfy Mach's notion of matter being necessary to define space). These three prejudices led Einstein to introduce the cosmological term to construct a static solution that was finite and yet had no boundaries—his universe curved back on itself like the surface of a balloon [see illustration on page 74]. Physically, the cosmological term would have been unobservable on the scale of our solar system, but it would

produce a cosmic repulsion on larger scales that would counteract the gravitational attraction of distant objects.  
sion and cause the universe to collapse, or will the cosmos expand forever? In the Friedmann models, the answer is tied to the average density of matter: a high-density universe will collapse, whereas a low-density universe will expand eternally. The dividing point is the critical-density universe, which expands forever albeit at an ever decreasing rate. Because, in Einstein's theory, the average curvature of the universe is tied to its average density, geometry and destiny are linked. The high-density universe is positively curved like

## In its current incarnation, the cosmological term arises not from relativity but from quantum mechanics.

field of strength  $g$  is exactly the same as the physics inside an elevator that is rocketing through empty space with a uniform acceleration of  $g$ .

Einstein was also strongly influenced by the philosophical notions of Austrian physicist Ernst Mach, who rejected the idea of an absolute frame of reference for spacetime. In Newtonian physics, inertia refers to the tendency of an object to move with constant velocity unless acted on by a force. The notion of constant velocity requires an inertial (that is, not accelerating) frame of reference. But not accelerating with respect to what? Newton postulated the existence of absolute space, an immovable frame of reference that defined all local inertial frames. Mach, though, proposed that the distribution of matter in the universe defined inertial frames, and to a large extent Einstein's general theory of relativity embodies this notion.

Einstein's theory was the first concept of gravity that offered a hope of providing a self-consistent picture of the whole universe. It allowed a description not only of how objects move through space and time but of how space and time themselves dynamically evolve. In using his new theory to try to describe the universe, Einstein sought a solution

that was finite, static and adhered to Mach's principles (for instance, a finite distribution of matter trailing off into emptiness did not seem to satisfy Mach's notion of matter being necessary to define space). These three prejudices led Einstein to introduce the cosmological term to construct a static solution that was finite and yet had no boundaries—his universe curved back on itself like the surface of a balloon [see illustration on page 74]. Physically, the cosmological term would have been unobservable on the scale of our solar system, but it would

produce a cosmic repulsion on larger scales that would counteract the gravitational attraction of distant objects.  
Einstein's enthusiasm for the cosmological term began to wane quickly, however. In 1917 Dutch cosmologist Willem de Sitter demonstrated that he could produce a spacetime solution with a cosmological term even in the absence of matter—a very non-Machian result. This model was later shown to be nonstatic. In 1922 Russian physicist Alexander Friedmann constructed models of expanding and contracting universes that did not require a cosmological term. And in 1930 British astrophysicist Arthur Eddington showed that Einstein's universe was not really static: because gravity and the cosmological term were so precariously balanced, small perturbations would lead to runaway contraction or expansion. By 1931, with the expansion of the universe firmly established by Hubble, Einstein formally abandoned the cosmological term as "theoretically unsatisfactory anyway."

Hubble's discovery obviated the need for the cosmological term to counteract gravity; in an expanding universe, gravity simply slows the expansion. The question then became, Is gravity strong enough to eventually stop the expan-

the surface of a balloon, the low-density universe is negatively curved like the surface of a saddle, and the critical-density universe is spatially flat. Thus, cosmologists came to believe that determining the universe's geometry would reveal its ultimate fate.

### The Energy of Nothing

THE COSMOLOGICAL TERM was banished from cosmology for the next six decades (except for a brief reappearance as part of the steady-state universe, a theory propounded in the late 1940s but decisively ruled out in the 1960s). But perhaps the most surprising thing about the term is that even if Einstein had not introduced it in a rush of confusion following his development of general relativity, we now realize that its presence seems to be inevitable. In its current incarnation, the cosmological term arises not from relativity, which governs nature on its largest scales, but from quantum mechanics, the physics of the smallest scales.

This new concept of the cosmological term is quite different from the one Einstein introduced. His original field equation,  $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ , relates the curvature of space,  $G_{\mu\nu}$ , to the distribution of matter and energy,  $T_{\mu\nu}$ , where  $G$  is Newton's constant characterizing



the strength of gravity. When Einstein added the cosmological term, he placed it on the left-hand side of the equation, suggesting it was a property of space itself [see box at right]. But if one moves the cosmological term to the right-hand side, it takes on a radically new meaning, the one it has today. It now represents a bizarre new form of energy density that remains constant even as the universe expands and whose gravity is repulsive rather than attractive.

Lorentz invariance, the fundamental symmetry associated with both the special and general theories of relativity, implies that only empty space can have this kind of energy density. Put in this perspective, the cosmological term seems even more bizarre. If asked what the energy of empty space is, most people would say "nothing." That is, after all, the only intuitively sensible value.

Alas, quantum mechanics is anything but intuitive. On the very small scales where quantum effects become important, even empty space is not really empty. Instead virtual particle-antiparticle pairs pop out of the vacuum, travel for short distances and then disappear again on timescales so fleeting that one cannot observe them directly. Yet their indirect effects are very important and can be measured. For example, the virtual particles affect the spectrum of hydrogen in a calculable way that has been confirmed by measurements.

Once we accept this premise, we should be prepared to contemplate the possibility that these virtual particles might endow empty space with some nonzero energy. Quantum mechanics thus makes the consideration of Einstein's cosmological term obligatory rather than optional. It cannot be dismissed as "theoretically unsatisfactory." The problem, however, is that all calculations and estimates of the magnitude of the empty-space energy lead to absurdly large values—ranging from 55 to 120 orders of magnitude greater than the energy of all the matter and radiation in the observable universe. If the vacuum energy density were really that high, all matter in the universe would instantly fly apart.

## THE COSMOLOGICAL TERM

### A Change of Meaning

The heart of Einstein's general theory of relativity is the field equation, which states that the geometry of spacetime ( $G_{\mu\nu}$ , Einstein's curvature tensor) is determined by the distribution of matter and energy ( $T_{\mu\nu}$ , the stress-energy tensor), where  $G$  is Newton's constant characterizing the strength of gravity. (A tensor is a geometric or physical quantity that can be represented by an array of numbers.) In other words, matter and energy tell space how to curve.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

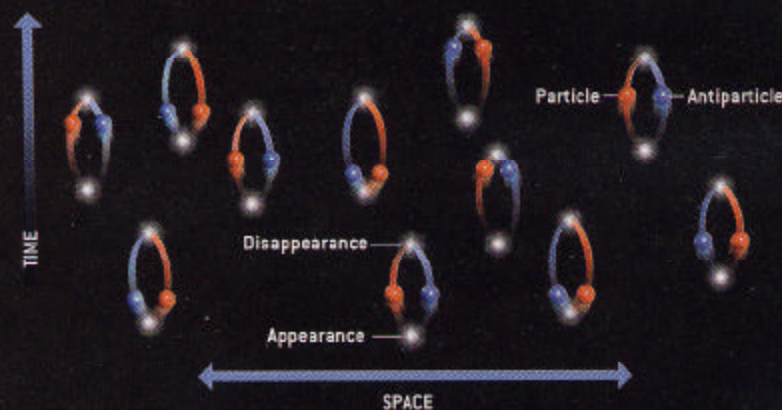
To create a model of a static universe, Einstein introduced the cosmological term  $\Lambda$  to counterbalance gravity's attraction on cosmic scales. He added the term (multiplied by  $g_{\mu\nu}$ , the spacetime metric tensor, which defines distances) to the left side of the field equation, suggesting that it was a property of space itself. But he abandoned the term once it became clear that the universe was expanding.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

The new cosmological term now being studied by physicists is necessitated by quantum theory, which holds that empty space may possess a small energy density. This term— $\rho_{\text{vac}}$ , the energy density of the vacuum, multiplied by  $g_{\mu\nu}$ —must go on the right side of the field equation with the other forms of energy.

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \rho_{\text{vac}} g_{\mu\nu})$$

Although Einstein's cosmological term and the quantum vacuum energy are mathematically equivalent, conceptually they could not be more different: the former is a property of space, the latter a form of energy that arises from virtual particle-antiparticle pairs. Quantum theory holds that these virtual particles constantly pop out of the vacuum, exist for a very brief time and then disappear (below).



#### THE AUTHORS

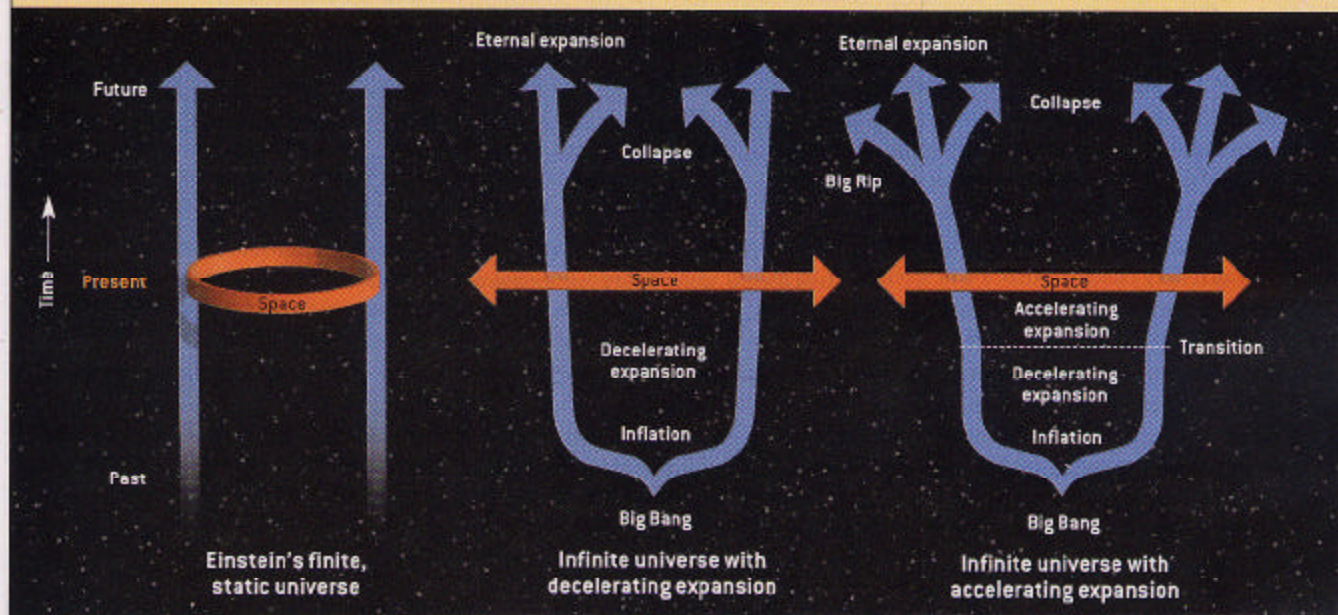
LAWRENCE M. KRAUSS and MICHAEL S. TURNER were among the first cosmologists to argue that the universe is dominated by a cosmological term that is radically different from the one introduced and then repudiated by Einstein. Their 1995 prediction of cosmic acceleration was confirmed by astronomical observations three years later. Chair of the physics department at Case Western Reserve University, Krauss has also written seven popular books, including *The Physics of Star Trek* and the soon-to-be-released *Hiding in the Mirror: The Mysterious Allure of Extra Dimensions*. Turner, who is Rauner Distinguished Service Professor at the University of Chicago, is now serving as the assistant director for mathematical and physical sciences at the National Science Foundation.



## Models of the Cosmos: Then and Now

Einstein's cosmological model (left) was a universe finite in space but infinite in time, remaining the same fixed size for eternity. This universe has no spatial boundaries; it curves back on itself like a circle. After the discovery of cosmic expansion, cosmologists constructed a model of an infinite universe in which the rate of expansion continuously slowed because of gravity (center), possibly leading to collapse. In the

1980s theorists added an early phase of rapid growth called inflation, for which there is now good evidence. In the past six years observations have shown that the cosmic expansion began to accelerate about five billion years ago (right). The ultimate fate of the universe—continued expansion, collapse or a hyper speedup called the big rip—depends on the nature of the mysterious dark energy driving the accelerated expansion.



This problem has been a thorn in the side of theorists for at least 30 years. In principle, it should have been recognized as early as the 1930s, when calculations of the effects of virtual particles were first performed. But in all areas of physics other than those related to gravity, the absolute energy of a system is irrelevant; what matters are the energy differences between states (for example, the energy differences between an atom's ground state and its excited states). If a constant is added to all the energy values, it drops out of such calculations, making it easy to ignore. Moreover, at that time few physicists took cosmology seriously enough to worry about applying quantum theory to it.

But general relativity implies that all forms of energy, even the energy of nothing, act as a source of gravity. Russian physicist Yakov Borisovich Zel'dovich realized the significance of this problem in the late 1960s, when he

made the first estimates of the energy density of the vacuum. Since that time, theorists have been trying to figure out why their calculations yield such absurdly high values. Some undiscovered mechanism, they reasoned, must cancel the great bulk of the vacuum energy, if not all of it. Indeed, they assumed that the most plausible value for the energy density is zero—even quantum nothingness should weigh nothing.

As long as theorists believed in the back of their minds that such a canceling mechanism might exist, they could place the cosmological term problem on the back burner. Although it was fascinating, it could be ignored. Nature, however, has intervened.

### Back with a Vengeance

THE FIRST DEFINITIVE evidence that something was amiss came from measurements of the slowing of the expansion rate of the universe. Recall that

Hubble found that the relative velocities of remote galaxies were proportional to their distance from our own galaxy. From the point of view of general relativity, this relation arises from the expansion of space itself, which should slow down over time because of gravitational attraction. And because very distant galaxies are seen as they were billions of years ago, the slowing of the expansion should lead to a curvature of the otherwise linear Hubble relation—the most distant galaxies should be receding faster than Hubble's law would predict. The trick, though, is accurately determining the distances and velocities of very remote galaxies.

Such measurements rely on finding standard candles—objects of known intrinsic luminosity that are bright enough to be seen across the universe. A breakthrough came in the 1990s with the calibration of type Ia supernovae, which are believed to be the thermonu-



clear explosions of white dwarf stars about 1.4 times the mass of the sun. Two teams—the Supernova Cosmology Project, led by Saul Perlmutter of Lawrence Berkeley National Laboratory, and the High-*z* Supernova Search Team, led by Brian Schmidt of Mount Stromlo and Siding Spring Observatories—set out to measure the slowing of the expansion of the universe using this type of supernova. In early 1998 both groups made the same startling discovery: over the past five billion years, the expansion has been speeding up, not

forms of matter—including cold dark matter, a putative sea of slowly moving particles that do not emit light but do exert attractive gravity—showed that matter contributes only about 30 percent of the critical density. A flat universe therefore requires some other form of smoothly distributed energy that would have no observable influence on local clustering and yet could account for 70 percent of the critical density. Vacuum energy, or something very much like it, would produce precisely the desired effect.

Einstein's closed universe, in which the density of the cosmological term was half that of matter.) Given the checked history of vacuum energy, our proposal was, at the very least, provocative.

A decade later, though, everything fits together. In addition to explaining the current cosmic acceleration and the earlier period of deceleration, a resurrected cosmological term pushes the age of the universe to almost 14 billion years (comfortably above the ages of the oldest stars) and adds exactly enough energy to bring the universe to the crit-

## Cosmological observations may illuminate the relation between gravity and quantum mechanics at a fundamental level.

slowing down [see "Cosmological Anti-gravity," by Lawrence M. Krauss; *SCIENTIFIC AMERICAN*, January 1999]. Since then, the evidence for a cosmic speedup has gotten much stronger and has revealed not only a current accelerating phase but an earlier epoch of deceleration [see "From Slowdown to Speedup," by Adam G. Riess and Michael S. Turner; *SCIENTIFIC AMERICAN*, February].

The supernova data, however, are not the only evidence pointing to the existence of some new form of energy driving the cosmic expansion. Our best picture of the early universe comes from observations of the cosmic microwave background (CMB), residual radiation from the big bang that reveals features of the universe at an age of about 400,000 years. In 2000, measurements of the angular size of variations of the CMB across the sky were good enough for researchers to determine that the geometry of the universe is flat. This finding was confirmed by a CMB-observing spacecraft called the Wilkinson Microwave Anisotropy Probe and other experiments.

A spatially flat geometry requires that the universe's average density must equal the critical density. But many different measurements of all

In addition, a third line of reasoning suggested that cosmic acceleration was the missing piece of the cosmological puzzle. For two decades, the paradigm of inflation plus cold dark matter has been the leading explanation for the structure of the universe. The theory of inflation holds that in its very first moments the universe underwent a tremendous burst of expansion that smoothed and flattened its geometry and blew up quantum fluctuations in energy density from subatomic to cosmic size. This event produced the slightly inhomogeneous distribution of matter that led to the variations seen in the CMB and to the observed structures in the universe today. The gravity of cold dark matter, which far outweighs ordinary matter, governed the formation of these structures.

By the mid-1990s, however, this paradigm was seriously challenged by observational data. The predicted level of matter clustering differed from what was being measured. Worse, the predicted age of the universe appeared to be younger than the ages of the oldest stars. In 1995 the two of us pointed out that these contradictions would disappear if vacuum energy accounted for about two thirds of the critical density. (This model was very different from

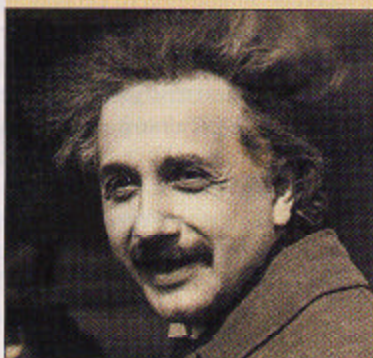
ical density. But physicists still do not know whether this energy actually comes from the quantum vacuum. The importance of discovering the cause of cosmic acceleration has brought a whole new urgency to the efforts to quantify vacuum energy. The problem of determining the weight of nothing can no longer be put aside for future generations. And the puzzle now seems even more confounding than it did when physicists were trying to devise a theory that would cancel vacuum energy. Now theorists must explain why vacuum energy might not be zero but so small that its effects on the cosmos became relevant only a few billion years ago.

Of course, nothing could be more exciting to scientists than a puzzle of this magnitude, richness and importance. Just as Einstein was led to general relativity by considering the incompatibility of special relativity and Newton's theory of gravity, physicists today believe that Einstein's theory is incomplete because it cannot consistently incorporate the laws of quantum mechanics. But cosmological observations may illuminate the relation between gravity and quantum mechanics at a fundamental level. It was the equivalence of accelerated frames and grav-

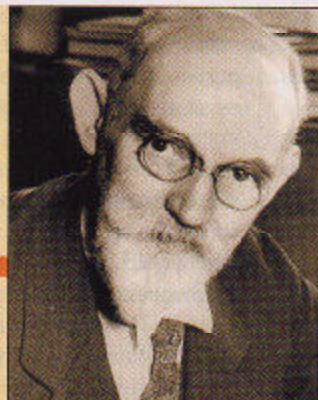


## A Checkered History

Since Einstein conceived the cosmological term almost 90 years ago, it has been repudiated, refashioned and resurrected. Here are some highlights.



**FEB. 1917:** Einstein introduces the cosmological term to counteract gravity, allowing him to build a theoretical model of a static, finite universe.



**MARCH 1917:** Dutch cosmologist Willem de Sitter produces an alternative model with a cosmological term. This model is later shown to have accelerating expansion.

**1922:** Russian physicist Alexander Friedmann constructs models of expanding and contracting universes without a cosmological term.



ity that pointed the way for Einstein; perhaps another kind of acceleration, the cosmic speedup, will point the way today. And theorists have already outlined some ideas about how to proceed.

### The Superworld

STRING THEORY, which is now often called M-theory, is viewed by many physicists as a promising approach to marrying quantum mechanics with gravity. One of the basic ideas underlying this theory is called supersymmetry, or SUSY. SUSY is a symmetry between

In the real world, however, we know that no selectron as light as the electron can exist because physicists would have already detected it in particle accelerators. (Theorists speculate that superpartner particles are millions of times heavier than electrons and thus cannot be found without the help of more powerful accelerators.) SUSY must therefore be a broken symmetry, which suggests that quantum nothingness might weigh something.

Physicists have produced models of broken supersymmetry yielding a vac-

served [see "The String Theory Landscape," by Raphael Bousso and Joseph Polchinski, on page 78].

Another hallmark of string theory is the positing of additional dimensions. Current theory adds six or seven spatial dimensions, all hidden from view, to the usual three. This construct offers another approach to explaining cosmic acceleration. Georgi Dvali of New York University and his collaborators have suggested that the effect of extra dimensions may show up as an additional term in Einstein's field equation

# The discovery of cosmic acceleration has forever altered our thinking about the future. Destiny is no longer tied to geometry.

particles of half-integer spin (fermions such as quarks and leptons) and those of whole-integer spin (bosons such as photons, gluons and other force carriers). In a world in which SUSY is fully manifest, a particle and its superpartner would have the same mass; for example, the supersymmetric electron (called the selectron) would be as light as the electron, and so on. In this superworld, moreover, it can be proved that quantum nothingness would weigh nothing and that the vacuum would have zero energy.

uum energy density that is many orders of magnitude smaller than the absurdly high estimates made previously. But even this theorized density is far larger than that indicated by cosmological observations. Recently, however, researchers have recognized that M-theory appears to allow for an almost infinite number of different solutions. Although almost all these possible solutions would indeed result in a vacuum energy that is far too high, some might produce a vacuum energy as low as the value that cosmologists have ob-

that leads to an accelerated expansion of the universe [see "Out of the Darkness," by Georgi Dvali; *SCIENTIFIC AMERICAN*, February]. This approach runs counter to long-held expectations: for decades, it had been assumed that the place to look for differences between general relativity and its successor theory would be at short distances, not cosmic ones. Dvali's plan flies in the face of this wisdom—if he is correct, the first harbinger of a new cosmic understanding will be at the largest distances, not the smallest.





**1929:** American astronomer Edwin Hubble discovers that the universe is expanding. Two years later Einstein abandons the cosmological term, calling it "theoretically unsatisfactory anyway."

**1967:** Russian physicist Yakov Borisovich Zel'dovich estimates the energy density of the quantum vacuum and finds that it would make an immense cosmological term.



**1998:** Two teams of supernova hunters led by Saul Perlmutter (left) and Brian Schmidt (right) report that the cosmic expansion is accelerating. A refashioned cosmological term would produce this effect. Since 1998 the evidence for cosmic acceleration has strengthened.

It is possible that the explanation of cosmic acceleration will have nothing to do with resolving the mystery of why the cosmological term is so small or how Einstein's theory can be extended to include quantum mechanics. General relativity stipulates that an object's gravity is proportional to its energy density plus three times its internal pressure. Any energy form with a large, negative pressure—which pulls inward like a rubber sheet instead of pushing outward like a ball of gas—will therefore have repulsive gravity. So cosmic acceleration may simply have revealed the existence of an unusual energy form, dubbed dark energy, that is not predicted by either quantum mechanics or string theory.

## Geometry vs. Destiny

IN ANY CASE, the discovery of cosmic acceleration has forever altered our thinking about the future. Destiny is no longer tied to geometry. Once we allow for the existence of vacuum energy or something similar, anything is possible. A flat universe dominated by positive vacuum energy will expand forever at an ever increasing rate [see illustration on page 70], whereas one dominated by negative vacuum energy will collapse. And if the dark energy is not vacuum energy at all, then its future impact on cosmic expansion is uncertain. It is possible that, unlike a cosmologi-

cal constant, the density of dark energy may rise or fall over time. If the density rises, the cosmic acceleration will increase, tearing apart galaxies, solar systems, planets and atoms, in that order, after a finite amount of time. But if the density falls, the acceleration could stop. And if the density becomes negative, the universe could collapse. The two of us have demonstrated that without knowing the detailed origin of the energy currently driving the expansion, no set of cosmological observations can pin down the ultimate fate of the universe.

To resolve this puzzle, we may need a fundamental theory that allows us to predict and categorize the gravitational impact of every single possible contribution to the energy of empty space. In other words, the physics of nothingness will determine the fate of our universe! Finding the solution may require new measurements of the cosmic expansion and of the structures that form

within it to provide direction for theorists. Fortunately, many experiments are being planned, including a space telescope dedicated to observing distant supernovae and new telescopes on the ground and in space to probe dark energy through its effect on the development of large-scale structures.

Our knowledge of the physical world usually develops in an atmosphere of creative confusion. The fog of the unknown led Einstein to consider a cosmological term as a desperate solution to constructing a static, Machian universe. Today our confusion about cosmic acceleration is driving physicists to explore every avenue possible to understand the nature of the energy that is driving the speedup. The good news is that although many roads may lead to dead ends, the resolution of this profound and perplexing mystery may eventually help us unify gravity with the other forces in nature, which was Einstein's fondest hope. ■

## MORE TO EXPLORE

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